Fachbereich C – Mathematik und Naturwissenschaften
– Physik –

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# Theoretical Solid State Physics, WS 08/09

## 10th practice sheet

Closing date: 08.01.2009, at 1:00 pm into the PO Box

22. **Debye Waller factor (6 points)**

The Debye Waller factor (DWF) for a simple cubic lattice is given by (cf the lecture):

$$W(q) = \frac{q^2\hbar}{4M}\int_0^\infty \frac{d\omega}{\omega} D(\omega) \coth\frac{\beta\hbar\omega}{2}, \tag{1}$$

where the density of states $D$ has the normalization $\int d\omega D(\omega) = 1$.

(a) Calculate the DWF for the Debye model (acoustic phonons) and analyse its behavior in the limits $k_BT \ll \hbar\omega_D$ and $k_BT \gg \hbar\omega_D$.

(b) Calculate the DWF for the Einstein model (optical phonons with frequency $\omega_0$) and analyse its behavior in the limits $k_BT \ll \hbar\omega_0$ and $k_BT \gg \hbar\omega_0$.

23. **Thermodynamics of the superconducting state (8 points)**

The equilibrium state of a superconducter in a uniform magnetic field is determined by the temperature $T$ and the magnitude of the field $H$. The thermodynamic identity is written in terms of the Gibbs free energy:

$$dG = -SdT - MdH, \tag{2}$$

where $S$ is the entropy and $M$ the total magnetization ($M = mV$, where $m$ is the magnetization density). The phase boundary between the superconducting and the normal states in the $H$-$T$ plane is given by the critical field curve $H_c(T)$ (cf the lecture).

(a) Deduce, from the fact that $G$ is continuous across the phase boundary, that

$$\frac{dH_c(T)}{dT} = \frac{S_n - S_s}{M_s - M_n}. \tag{3}$$

(b) Using the fact that the superconducting state displays perfect diamagnetism ($B = 0$), while the normal state has negligible diamagnetism ($M \approx 0$), show that the entropy discontinuity across the phase boundary is

$$S_n - S_s = -\frac{V}{4\pi} H_c \frac{dH_c}{dT} \tag{4}$$

and thus the latent heat, when the transition occurs in a field, is

$$Q = -TV\frac{H_c}{4\pi}\frac{dH_c}{dT}. \tag{5}$$

(c) Show that when the transition occurs at zero field there is a specific heat discontinuity given by

$$(c_p)_n - (c_p)_s = -\frac{T}{4\pi}\left(\frac{dH_c}{dT}\right)^2. \tag{6}$$