Fachbereich C – Mathematik und Naturwissenschaften
– Physik –

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Theoretical Solid State Physics, WS 08/09

9th practice sheet

Closing date: 19.12.2008, at 1:00 into the PO Box

1. **Density of states (8 points)**

   Calculate the density of states $D(\omega)$ in the following cases:

   (a) the monoatomic harmonic chain (lattice constant a, mass m, spring constant k) with dispersion relation
   $$\omega^2(q) = 4\frac{k}{m}\sin^2(qa/2), \qquad -\pi \le q < \pi; \tag{1}$$

   (b) the diatomic harmonic chain (lattice constant a, masses m and M, spring constant k) with dispersion relation
   $$\omega^2(q) = \frac{k}{m} + \frac{k}{M} \pm k\sqrt{\left(\frac{1}{m}+\frac{1}{M}\right)^2 - \frac{4\sin^2(qa)}{mM}}, \qquad -\pi \le q < \pi; \tag{2}$$

   (c) the acoustic branch in d Dimensions in the Debye-model, i.e. $\omega(\vec{q}) = c|\vec{q}|$.

   Plot $D(\omega)$ in part (a) and (b) and for $d = 1, 2, 3$ in part (c).

2. **BCH formula (6 points)**

   (a) Let A and B two operators fulfilling $[A,[A,B]] = [B,[A,B]] = 0$. Proof the Baker-Campell-Hausdorff formula:
   $$e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,b]} \tag{3}$$
   Hint: Define the operators $f(x) := e^{Ax}e^{Bx}$, $g(x) := e^{Ax}Be^{-Ax}$. Consider $f'(x)$, $g'(x)$ and higher derivations of g and solve the differential equations of first order for f and g with respect to appropriate initial conditions.

   (b) Show for an operator L, linear in the creation and annihilation operators b and $b†$ of a harmonic oscillator, i.e. $L = xb + yb^\dagger$, the following relation:
   $$\langle e^L \rangle = e^{\frac{1}{2}\langle L^2 \rangle}, \tag{4}$$
   where the brackets $\langle\cdot\rangle$ describe the expectation value of the canonical density operator of the harmonic oscillator ($H = \epsilon(b^\dagger b + \frac{1}{2})$).
   Hint: Use the BCH formula to get an expectation value of the form $h(x) = \langle e^{xb}e^{xb^\dagger}\rangle$. Derive a differential equation for h and solve it.