Fachbereich C – Mathematik und Naturwissenschaften
– Physik –

Prof. Dr. A. Klümper
M. Brockmann
(G-16.04, 439-2541, michael.brockmann@physik.uni-wuppertal.de)

Theoretical Solid State Physics, WS 08/09

7th practice sheet

Closing date: 04.12.2008, at 1:00 pm into the PO Box

16. **Harmonic oscillations of a two-dimensional lattice (8 points)**

    Consider a two-dimensional square lattice with periodic boundary conditions composed of identical ions with mass m. Every ion interacts with its nearest and next-nearest neighbours. The spring constants of the harmonic potential are given by β_1 for nearest neighbours and β_2 for next-nearest neighbours. All other interactions are negligible. Furthermore all motions of the ions are confined to the lattice plane. Set up the coefficient matrix $(G_{na,mb})$ of the harmonic potential and compute the dynamical Matrix $G(\vec{q})$. Calculate the solution of the equations of motion via diagonalization of $G(\vec{q})$. How does the frequency depend on the wave vector $\vec{q}$? Plot the dispersion relation for the $(q, 0)$- and the (q, q)-directions.

17. **One dimensional lattice with a two-atomic basis (8 points)**

    Consider a one-dimensional Bravais lattice with two ions per unit cell, with equilibrium positions na and $na + d$. We take the two ions to be identical, but take $d \leq a/2$. For simplicity we assume that only nearest neighbours interact. As a consequence the force between neighbouring ions depends on whether the distance is d or $a - d$. The harmonic potential energy can be written in the form:

    $$U = \frac{K}{2} \sum_n (u_1(na) - u_2(na))^2 + \frac{G}{2} \sum_n (u_2(na) - u_1(na + a))^2, \qquad (1)$$

    where $u_1(na)$ $(u_2(na))$ is the displacement of the ion which oscillates about the site na $(na + d)$. Due to $d \leq a/2$ we have the relation $K \geq G$.
    Calculate the equations of motion and the dispersion relation assuming periodic boundary conditions. Analyse this relation!