Fachbereich C – Mathematik und Naturwissenschaften
– Physik –

Prof. Dr. A. Klümper
M. Brockmann
(G-16.04, 439-2541, michael.brockmann@physik.uni-wuppertal.de)

Theoretical Solid State Physics, WS 08/09

6th practice sheet

Closing date: 27.11.2008, at 13:00 p.m. into the PO Box

14. **Magnetoresistance I (6 points)**

A conductor is exposed to a magnetic field $\vec{B} = B\vec{e}_z$. Calculate the magnetoresistance $\rho(B) = \frac{\vec{j}\cdot\vec{E}}{j^2}$ using the equations (2) and (3) of the 4th practice sheet for the following cases of different energy bands:

(a) Closed surfaces of constant energy: $E_n(\vec{k}) = \frac{\hbar^2\vec{k}^2}{2m}$ (approximation of free electrons) and $\omega_c\tau \gg 1$, where $\omega_c = |e|B/mc$ is the cyclotron frequency and τ is the relaxation time.

(b) Open surfaces of constant energy: $E_n(\vec{k}) = \frac{\hbar^2(\vec{n}\cdot\vec{k})^2}{2m}$ (complete flat energy bands with normal vector $\vec{n}$). Show that $\vec{v} = -\frac{e\tau}{m}(\vec{n}\cdot\vec{E})\vec{n}$. Argue that, as a consequence, ρ must be infinite except for $\vec{j}||\vec{n}$.

(c) Open surfaces of constant energy which are almost flat. Argue that the $\vec{j}$-$\vec{E}$-relation is given by $\vec{j} = (\sigma^{(0)}+\sigma^{(1)})\vec{E}$, where $\sigma^{(0)} = \frac{ne^2\tau}{m}\left(\vec{n}^T\otimes\vec{n}\right)$ and $\sigma^{(1)}$ is a small correction tensor which vanishes for $B\to\infty$. Invert this relation where $\vec{n} = \vec{e}_x$ and $\sigma^{(1)}$ is assumed to be diagonal. Show that $\rho\to\infty$ if $B\to\infty$.

15. **Magnetoresistance II (8 points)**

(a) Verify the following equivalence relation for $\vec{y}\cdot\vec{z} = 0$:

$$\vec{y} = \frac{1}{1+z^2}(\vec{x}+\vec{z}\times\vec{x}) \qquad \Leftrightarrow \qquad \vec{x} = \vec{y}-\vec{z}\times\vec{y} \tag{1}$$

(b) Does the geometry of Hall's experiment show magnetoresistance $\rho := \frac{\vec{j}\cdot\vec{E}}{j^2}$ with non-trivial dependence on the magnetic field B? Hint: Set $\vec{x} = \vec{E}$, $\vec{y} = \vec{j}/\sigma_0$, $\vec{z} = \gamma\vec{B}/B$, $\gamma = \omega_c\tau$ and check the requirements of part (a).

(c) Consider now a system with more than one type of charge carriers:

$$\vec{E} = \frac{\vec{j}_k}{\sigma_k} + \frac{\gamma_k}{\sigma_k}(\vec{B}/B\times\vec{j}_k), \qquad k = 1,\ldots,n. \tag{2}$$

What is the physical background of such a system? Calculate $\rho(B) = (\vec{j}\cdot\vec{E})/j^2$ with $\vec{j} = \sum_k \vec{j}_k$. Note that the dependence of B comes from the γ_k's.

(d) Calculate $\rho_0 := \rho(0)$, $\rho(B)$ for $B\to\infty$ and $\rho(B)$ if all γ_k's are equal.

(e) A two-band metal ($n=2$) is called compensated if $\gamma_1/\sigma_1 = -\gamma_2/\sigma_2$, i.e. $R_{H,1} = -R_{H,2}$. Show that in this case the magnetoresistance increases as B^2 with increasing field B.