Fachbereich C – Mathematik und Naturwissenschaften
– Physik –

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Theoretical Solid State Physics, WS 08/09

4th practice sheet

Closing date: 13.11.2008, at 12:00 into the PO Box

10. **Dynamics of band electrons (4 points)**

The Hamiltonian of an electron in energy band n exposed to an electro-magnetic field is given by

$$H_n = E_n(\vec{p} - e/c\,\vec{A}) + e\Phi(\vec{r}). \tag{1}$$

Show the relations

$$\dot{\vec{r}} = \vec{\nabla}_{\vec{k}} E_n(\vec{k}), \tag{2}$$

$$\dot{\vec{k}} = \frac{e}{c}\left(\vec{\nabla}_{\vec{k}} E_n(\vec{k}) \times \vec{B}\right) + e\vec{E}. \tag{3}$$

Hint: Prove first that $[k_\alpha, k_\beta] = i\frac{e}{c}\,\varepsilon_{\alpha\beta\gamma} B_\gamma$.

11. **Landau quantization (6 points)**

Consider a free electron with mass m and charge e in a magnetic field $\vec{B} = B\vec{e}_z$. The Hamiltonian H is given by

$$H = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2. \tag{4}$$

The Landau gauge $\vec{A} := Bx\vec{e}_y$ fulfils $\vec{B} = B\vec{e}_z$ and $\vec{p}\vec{A} = \vec{A}\vec{p}$.

(a) Show that the energy eigenvalues are given by

$$E_{n,k_y,k_z} = \hbar\omega_0\left(n + \frac{1}{2}\right) + \frac{\hbar^2 k_z^2}{2m}, \tag{5}$$

where $\omega_0 = |e|B/mc$ is called the cyclotron frequency.
Hint: Show that the Hamiltonian transforms into

$$H = \frac{p_x^2}{2m} + \frac{m}{2}\omega_0^2\left(x - \frac{p_y}{m\omega_0}\right)^2 + \frac{p_z^2}{2m} \tag{6}$$

and that the ansatz $\Psi(\vec{r}) = \phi(x)e^{ik_y y}e^{ik_z z}$ leads to the Schrödinger equation for the one-dimensional harmonic oscillator. How does the solution depend on the wavenumber k_y?

(b) Determine the degree of degeneracy. Assume periodic boundary conditions in y-direction: $k_y = 2\pi l_y/L_y$, $l_y \in \mathbb{N}$, and use the condition, that the midpoint of the oscillator x_0 is resricted to $0 \le x_0 \le L_x$.

12. **Fermi surface and rotational frequency (4 points)**

In the lecture was the following formula discussed:

$$\frac{2\pi}{\omega_c} = \frac{c}{|e|B}\frac{dS}{dE}, \tag{7}$$

where ω_c is the cyclotron frequency and $S(E)$ the surface of the area bounded by the orbit (for details confer the lecture). Prove this equation!