Exercise 1 for Theoretical Solid State Physics in WS 2012

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1. **Quantum Mechanics (15 Points)**

   (a) The Stationary Schroedinger Equation is

   $$\mathbf{H}\left(\{\vec{\mathbf{r}}_j, \vec{\mathbf{p}}_j\}\right)\psi\left(\{\vec{r}_j\}\right) = E\psi\left(\{\vec{r}_j\}\right).$$

   Show that you can write this equation in one dimension for a particle of mass m in a potential $V(x)$ as

   $$\psi''(x) = \left(u(x) - \epsilon\right)\psi(x), \qquad \epsilon = \frac{2m}{\hbar^2}E, \qquad u(x) = \frac{2m}{\hbar^2}V(x). \tag{1}$$

   (b) Solve equation (1) for an incoming wave from the left with $\epsilon > u(x) = 0$.

   (c) Solve equation (1) for

   $$u(x) = \begin{cases} 0 & -d \le x \le d \\ u & \text{otherwise} \end{cases}, \qquad d,\, u > 0.$$

   Consider explicitly the cases $0 < \epsilon < u$, $\epsilon = u$ and $\epsilon > u$. Which equations do you get, if you use the continuity conditions?

   (d) Consider the potential

   $$u(x) = \begin{cases} 0 & -d \le x \le 0 \\ u > 0 & 0 \le x \le d \\ \infty & \text{otherwise} \end{cases}.$$

   i. Solve equation (1) for all 3 cases. Which equations do you get, if you use the continuity conditions?

   ii. Sketch qualitatively the solutions for $0 < \epsilon < u$ and $\epsilon > u$.

   iii. Now let $d = 3$ and $u = 10$. How many solutions exist for $0 < \epsilon < u$ and what are the energy eigenvalues for ϵ.

2. **Face centered tetragonal structure (2 Points)**
   The tetragonal face-centered structure do not appear explicitly in the list of the 14 Bravais lattices. Why? How can you describe these lattices as one of the 14 Bravais lattices?

3. **Symmetries of unit cells (3 Points)**
   Find all symmetry axes and planes of the simple cubic lattice and the hexagonal Bravais lattice.