# Exercise 8 for Theoretical Solid State Physics in Summer 2023

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1. **Born-Oppenheimer approxiation (8 points)**
   A heavy particle $M$ and a light particle $m \ll M$ move inside an infinitely high potential well of width $L$. The particles experience an attractive potential interaction described by the potential $W(r-R) = -\lambda\delta(r-R)$, where $\lambda > 0$ and $R$ and $r$ are the positions of the heavy and light particle, respectively.
   Calculate the spectrum of the corresponding Hamiltonian

$$H = -\frac{\hbar^2}{2m}\partial_r^2 - \frac{\hbar^2}{2M}\partial_R^2 - \lambda\delta(r-R)$$

   in Born-Oppenheimer-approximation.

   (a) In step one transform the Hamiltonian into a dimensionless form and neglect the kinetic energy of the heavy particle. Determine the unnormalized eigenfunctions and an equation for the eigenvalues $\epsilon_n(R)$ of the light particle. Consider only the case $\epsilon > 0$.

   (b) What is the meaning of $\epsilon_n(R)$ and how should we include qualitatively the heavy particle into our picture?

2. **Density of states (8 points)**
   Calculate and plot (for $d = 1, 2, 3$ in part (c)) the density of states $D(\omega)$ in the following cases:

   (a) the monoatomic harmonic chain (lattice constant $a$, mass $m$, spring constant $k$) with dispersion relation

$$\omega^2(q) = \frac{4k}{m}\sin^2\frac{qa}{2}, \qquad -\frac{\pi}{a} \le q < \frac{\pi}{a},$$

(b) the diatomic harmonic chain (lattice constant $a$, masses $m$ and $M$, spring constant $k$) with dispersion relation

$$\omega^2(q) = \frac{k}{m} + \frac{k}{M} \pm k\sqrt{\left(\frac{1}{m} + \frac{1}{M}\right)^2 - \frac{4}{mM}\sin^2(qa)}, \qquad -\frac{\pi}{a} \leq q < \frac{\pi}{a},$$

(c) the acoustic branch in $d$ dimensions in the Debye model, i. e. $\omega(\vec{q}) = e\,|\vec{q}|$.