Exercise 6 for Theoretical Solid State Physics in Summer 2021

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Submission: 21.05.2021, 12:00 in the P.O. Box Giesen on D.10 or by e-mail
Discussion: 24.05.2021, 14:15

1. **Density of states (5 points)**

   In the lecture we have considered free particles with energy momentum dispersion $E(\vec{k}) = \vec{k}^2/2m$ in d-dimensions. We computed the density of states where we focused on $d = 1, 2, 3$.

   Now let us assume that an energy momentum dispersion $E(\vec{k}) = v|\vec{k}|$ in d-dimensions is given. In other words, the energy is linear in the absolute value of the momentum $\vec{k}$.

   (a) Compute the density of states and plot these for the cases $d = 1, 2, 3$.

   (b) Which (fermionic) particles have a linear dispersion law?

   (c) Have you heard about electrons on certain (2d) lattices with "linear regions" of the energy bands?

2. **Van-Hove singularities (12 points)**

   The density of states $D_n(E)$ of the n-th energy band is given by

$$D_n(E) = \frac{1}{(2\pi)^3} \int\limits_{E_n(\vec{k})=E} \frac{\mathrm{d}S}{\left|\vec{\nabla}_{\vec{k}} E_n\left(\vec{k}\right)\right|}.$$

   The total density of states is the sum over all band indices n. Analyse the different types of singularities. Expand the energies close to the critical point E_0

$$E_n\left(\vec{k}\right) = E_0 + \left(\frac{k_x^2}{2m_{xn}^*} + \frac{k_y^2}{2m_{yn}^*} + \frac{k_z^2}{2m_{zn}^*}\right) + \mathcal{O}\left(k^4\right)$$

   and discuss the four different cases of relative signs of m_{jn}^*, $j = x, y, z$.