Exercise 1 for Theoretical Solid State Physics in Summer 2021

Prof. Dr. Andreas Klümper
Mathis Giesen (jan.giesen@uni-wuppertal.de G.11.07)

Submission: 19.04.2021, 12:00 in the P.O. Box Giesen on D.10
Discussion: ??.04.2021, ??:??

1. **Quantum Mechanics (15 Points)**

   (a) The Stationary Schrödinger Equation is

   $$\mathbf{H}\psi = E\psi.$$

   Show that you can write this equation in one dimension for a particle of mass m in a potential $V(x)$ as

   $$\psi''(x) = \frac{2m}{\hbar^2}\left(V(x) - E\right)\psi(x). \tag{1}$$

   (b) Solve equation (1) for an incoming wave from the left with $E > V(x) = 0$.

   (c) Solve equation (1) for

   $$V(x) = \begin{cases} 0 & -d \leq x \leq d \\ V & \text{otherwise} \end{cases}, \qquad d,\, u > 0.$$

   Consider explicitly the cases $0 < E < V$, $E = V$ and $E > V$. Which equations do you get if you use the continuity conditions?

   (d) Consider the potential

   $$V(x) = \begin{cases} 0 & -d \leq x \leq 0 \\ V > 0 & 0 \leq x \leq d \\ \infty & \text{otherwise} \end{cases}.$$

   i. Solve equation (1) for all three cases. Which equations do you get, if you use the continuity conditions?

ii. Sketch qualitatively the solutions for $0 < E < V$ and $E > V$.

iii. Now let $d = 3$ and $\frac{2m}{\hbar^2}V = 10$. How many solutions exist for $0 < E < V$ and what are the energy eigenvalues for $\frac{2m}{\hbar^2}E$.