FQM problem Sheet 9 in WS 2022/2023

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1. **U(1) symmetry and charge conservation** (10)
   The Lagrangian densities $\mathcal{L}_{KG} = \frac{1}{2}(\partial_\mu\bar{\varphi}\partial^\mu\varphi - m^2\bar{\varphi}\varphi)$ and $\mathcal{L}_D = \bar{\psi}(\not{p} - m)\psi$ are invariant under the transformation $\varphi \to e^{-i\varepsilon}\varphi$, $\bar{\varphi} \to e^{i\varepsilon}\bar{\varphi}$.
   (a) **Derive** by use of Noether's theorem the associated current densities and charges $\mathcal{Q}$.
   (b) **Calculate** the commutator (or Poisson bracket) of $\mathcal{Q}$ and φ. Why does the operator $\mathcal{Q}$ determine the charge in quantum field theory?
   (c) Does the real KG density also have such a conserved current?

2. **Energy-momentum tensor** (11)
   Calculate the energy-momentum tensor
   $$\mathcal{J}_{\mu\nu} = -g_{\mu\nu}\mathcal{L} + \sum_r \frac{\partial\mathcal{L}}{\partial(\partial\varphi_r/\partial x_\mu)}\frac{\partial\varphi_r}{\partial x^\nu}$$
   (a) for the Maxwell Lagrangian density $\mathcal{L}_M = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$.
   (b) for the Klein-Gordon Lagrangian density $\mathcal{L}_{KG} = \frac{1}{2}(\partial_\mu\varphi\partial^\mu\varphi - m^2\varphi^2)$.
   (c) for the Dirac Lagrangian density $\mathcal{L}_D = \bar{\psi}(\not{p} - m)\psi$.