FQM problem Sheet 8 in WS 2022/2023

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Discussion of sheet 7 and 8: 07.12.2022, 10:15 – 11:45

1. **Classical field theory** (10)
   Consider the electromagnetic potential $\boldsymbol{A} := (A^\mu) = (\Phi, \vec{A})$

   (a) **Calculate** the components $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ of the electromagnetic field strength tensor.

   (b) Use the Lagrangian density $\boldsymbol{\mathcal{L}}_f(A^\mu, \partial_\nu A^\mu) = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$ of the free electromagnetic field.

      i. Set up the Euler-Lagrange equations. (What do you notice?)

      ii. **Calculate** the Hamiltonian function $\boldsymbol{H} = \int d^3x\, \boldsymbol{\mathcal{H}}$ by deriving the Hamiltonian density $\boldsymbol{\mathcal{H}}$ from the Lagrangian density using the Legendre transform. **Remember**: $\boldsymbol{\mathcal{H}} = \sum_{k=1}^{3} \pi^k \dot{A}_k - \boldsymbol{\mathcal{L}}$, where π^k are the canonically conjugate momenta to A^k.

      iii. Can you devise an additional term to the Lagrangian density such that Maxwell's inhomogeneous equations with source terms ($j^\mu \neq 0$) arise?