FQM problem Sheet 7 in WS 2022/2023

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This problem sheet is rather short. So possibly in the afternoon of 30.11. instead of the tutorial we will have another lecture and the next tutorial will be on 07.12.

1. **The Klein-Gordon field** (8)
   The Klein-Gordon field is given as

$$\varphi(x,t) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}}[a(k)e^{i(k\cdot x-\omega_k t)} + a^\dagger(k)e^{-i(k\cdot x-\omega_k t)}] = \int d^3k\,[a(k)f_k(x,t) + a^\dagger(k)f_k^*(x,t)]$$

   with $\omega_k = +\sqrt{k^2+m^2}$, bosonic $a(k)$, $a^\dagger(k)$ satisfying standard commutation relations, and $f_k(x,t)$ is a short-hand for the plane wave with an obvious scale factor.

   (a) **Show**

$$\int d^3x\, f_k^*(x,t)\varphi(x,t) = \frac{1}{2\omega_k}[a(k) + a^\dagger(-k)e^{2i\omega_k t}]$$

   (b) **Show**

$$\int d^3x\, f_k^*(x,t)\dot{\varphi}(x,t) = \frac{-i}{2}[a(k) - a^\dagger(-k)e^{2i\omega_k t}]$$

   (c) **Derive** from (a) and (b)

$$a(k) = i\int d^3x\, f_k^*(x,t)\overset{\leftrightarrow}{\partial_0}\varphi(x,t)$$

   where $a(t)\overset{\leftrightarrow}{\partial_0} b(t) = a(t)\dot{b}(t) - \dot{a}(t)b(t)$.