FQM problem Sheet 10 in WS 2022/2023

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Discussion: 21.12.2022, 14:00 – 16:00

1. **Spin-1 wave equation** (10)
   The wave equation for a spin-1 particle of mass m with a 4-component field is given as

   $$\left[g_{\mu\nu}(\Box + m^2) - \partial_\mu\partial_\nu\right]\varphi^\nu(x) = 0.$$

   (a) **Show** $\partial_\nu\varphi^\nu = 0$.

   (b) **Derive** the following Lagrangian density:

   $$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi^\nu\partial^\mu\varphi_\nu + \frac{m^2}{2}\varphi_\nu\varphi^\nu + \frac{1}{2}(\partial_\nu\varphi^\nu)^2$$

   (c) **Derive** the Hamilton density

   $$\mathcal{H} = -\frac{1}{2}\pi_\mu\pi^\mu - \frac{1}{2}(\nabla\varphi_\mu)\cdot\nabla\varphi^\mu - \frac{m^2}{2}\varphi_\mu\varphi^\mu$$

   where $\pi_\mu = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}^\mu}$.

   (d) **Show** that the Hamiltonian equations of motion with the constraint $\pi_0 = \partial_i\varphi^i = -\dot{\varphi}_0$ yield the original wave equation.

2. **Green's function of the Dirac equation** (12)
   We consider the Feynman propagator $S_F(x-x')_{\beta\alpha} := -i\langle 0|\boldsymbol{\mathcal{T}}(\psi_\beta(x)\cdot\bar{\psi}_\alpha(x'))|0\rangle$.

   (a) **Show**

   $$S_F(x-x') = \lim_{\varepsilon\to 0}\int\frac{d^4k}{(2\pi)^4}e^{-ik\cdot(x-x')}\frac{\not{k}+m}{k^2-m^2+i\varepsilon}$$

   by

   i. calculating the left hand side using the definition of the Dirac field operators,
   ii. simplifying the right hand side using the residue theorem.

   Now why does $(i\not{\partial} - m)S_F(x-x') = \delta^4(x-x')$ hold?