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Ex 3: Yang-Baxter relations for rational fusion-type R-matrices

Let P be the transposition matrix acting on $\mathbb{C}^2 \otimes \mathbb{C}^2$ and

$$R(\lambda) = \frac{\lambda + \mathrm{i}P}{\lambda + \mathrm{i}}$$

the corresponding rational solution of the Yang-Baxter equation

$$R_{12}(\lambda - \mu) R_{13}(\lambda) R_{23}(\mu) = R_{23}(\mu) R_{13}(\lambda) R_{12}(\lambda - \mu)\,.$$

Let $S : \mathbb{C}^2 \otimes \mathbb{C}^2 \to \mathbb{C}^3$ be defined by the matrix

$$S = \begin{pmatrix} 1 & & & \\ & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \\ & & & 1 \end{pmatrix}\,.$$

In the lecture we set $R^{[\frac{1}{2}\frac{1}{2}]}(\lambda) = R(\lambda)$ and introduced the fused R-matrices

$$\begin{aligned} R^{[\frac{1}{2}1]} &= S_{23} R_{13}(\lambda) R_{12}(\lambda + \mathrm{i}) S_{23}^t\,, \\ R^{[1\frac{1}{2}]} &= S_{12} R_{13}(\lambda - \mathrm{i}) R_{23}(\lambda) S_{12}^t\,, \\ R^{[11]} &= S_{12} S_{34} R_{14}(\lambda - \mathrm{i}) R_{13}(\lambda) R_{24}(\lambda) R_{23}(\lambda + \mathrm{i}) S_{34}^t S_{12}^t\,. \end{aligned}$$

We claimed that they satisfy the Yang-Baxter relations

$$R_{12}^{[s_1 s_2]}(\lambda - \mu) R_{13}^{[s_1 s_3]}(\lambda) R_{23}^{[s_2 s_3]}(\mu) = R_{23}^{[s_2 s_3]}(\mu) R_{13}^{[s_1 s_3]}(\lambda) R_{12}^{[s_1 s_2]}(\lambda - \mu)$$

for $s_j \in \{\frac{1}{2}, 1\}$, $j = 1, 2, 3$, but we only proved the cases $(s_1, s_2, s_3) = (\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, $(\frac{1}{2}, \frac{1}{2}, 1), (\frac{1}{2}, 1, 1), (1, 1, 1)$. Prove the remaining four cases!

(8 points)

Ex 4: Explicit form of a rational fusion-type R-matrix

Obtain $R^{[1\frac{1}{2}]}(\lambda)$ defined in the previous exercise explicitly as a 6×6 matrix!

(4 points)